Engineering Notes

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Ordinary Coherence Functions and Mechanical Systems

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Nomenclature

$C_{xy}(f)$ = one-sided co-spectrum
$G_x(f)$ = power spectral density function
$G_{xy}(f)$ = one-sided cross-spectral density function
$G_{xy}^*(f)$ = conjugate of $G_{xy}(f)$
$H(f)$ = system frequency response function
$Q_{xy}(f)$ = one-sided quad-spectrum
$R_{xy}(\tau)$ = cross-correlation function between the input and system response
$\theta_{xy}(f)$ = phase angle associated with the system frequency response function
$\gamma_{xy}^2(f)$ = ordinary coherence function

Introduction

IN this Note, application of ordinary coherence functions to mechanical systems excited by random exitation is discussed and experimental results are shown for a single input—single output viscoelastic beam configuration.

Coherence functions in random data analysis are well established in theory.[1-5] To date, practical application is found principally in seismic data analysis[6] and related fields.[7,8] Some work is reported for mechanical systems.[9] Such functions are used in assessing the validity of frequency response functions from measured random data, in eliminating redundant data channels, and in potentially detecting system nonlinearities. Their use, therefore, appears well suited to random vibration problems of missiles and spacecraft; in particular to problems of random testing and dynamic environment predictions.

Single Input—Single Output Relationships

For a single input—single output configuration, an ordinary coherence function is[1]

$$\gamma_{xy}^2(f) = \frac{G_{xy}(f)G_{xy}^*(f)}{G_x(f)G_y(f)} = \frac{|G_{xy}(f)|^2}{G_x(f)G_y(f)} \tag{1}$$

and ranges in magnitude

$$0 \leq \gamma_{xy}^2(f) \leq 1, \text{ for } 0 \leq f \leq \infty$$

The one-sided cross-spectral density $G_{xy}(f)$ is the complex function

$$G_{xy}(f) = C_{xy}(f) - iQ_{xy}(f) \tag{2}$$

where

$$C_{xy}(f) = 2\int_0^\infty [R_{xy}(\tau) + R_{yx}(\tau)] \cos 2\pi f\tau d\tau$$
$$Q_{xy}(f) = 2\int_0^\infty [R_{xy}(\tau) - R_{yx}(\tau)] \sin 2\pi f\tau d\tau \tag{3}$$

since $R_{yx}(\tau) = R_{xy}(-\tau)$. In polar form,

$$G_{xy}(f) = |G_{xy}(f)|e^{-\theta_{xy}(f)} \tag{4}$$

where

$$|G_{xy}(f)| = [C_{xy}^2(f) + Q_{xy}^2(f)]^{1/2}$$
$$\theta_{xy}(f) = \tan^{-1}[Q_{xy}(f)/C_{xy}(f)] \tag{5}$$

Now for a linear system,

$$G_y(f) = |H(f)|^2G_x(f) \tag{6}$$

$$G_{xy}(f) = H(f)G_x(f) \tag{7}$$

and, upon substituting into Eq. (1),

$$\gamma_{xy}^2(f) = 1 \tag{8}$$

This implies that the input $x(t)$ and the output $y(t)$ are related linearly over all time at all frequencies. However, if the system 1) is nonlinear, 2) has multiple-input excitation, or 3) experiences extraneous signals (such as noise) at either $x(t)$ or $y(t)$ or both, then $\gamma_{xy}^2(f) < 1$. It is clear that if $\gamma_{xy}^2(f)$ is to be used as a measure of system linearity, then the last two conditions somehow must be accounted for in the computation defined by Eq. (1).

Discussion of Experimental Results

To assess the practicality of such single input—single output concepts, a cantilevered viscoelastic beam shown by Fig. 1 was base excited in the laboratory by 1) simple harmonic excitation and 2) band-limited white noise. By conventional sine dwells, the harmonic excitation was used to determine the frequency response function between the tip response acceleration and the base input acceleration (i.e., the magnification factor). For the random input, the base and tip acceleration signals were recorded on magnetic tape, digitized, and processed to form the magnification factor and the companion coherence function. Comparisons then were made of the two magnification factors.

The test specimen consisted of layered viscoelastic damping material (Lord, LD-400) bonded between two 0.0625 in. steel strips. In the fundamental mode, the damping factor was approximately $\zeta = 0.05$. The frequency range of the

Table 1 Data reduction parameters

Parzen lag window[10]	
Record length	5.94 sec
Resolution	4.88 cycles
Samples/sec	2500
Degrees of freedom	58

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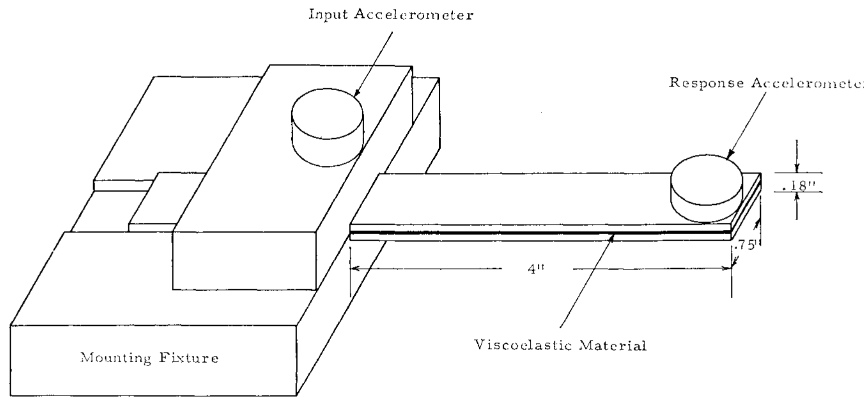

Fig. 1 Sketch of beam configuration.

accelerometers was 20 Hz to 3000 Hz, and the data were recorded in a multiplexed manner. The harmonic excitation consisted of 10 sec, $0.5g_{rms}$ sine dwells. As shown by Fig. 2, the input random acceleration had an essentially flat spectrum over the frequency range 20 Hz to 3000 Hz with an over-all test level of $2.83g_{rms}$. These input levels allowed the output response to be both linear and convenient for measurement. Facts pertinent to all data reduction are shown in Table 1.

Figure 3 depicts the acceleration (ordinary) spectral density of the response where the over-all $g_{rms} = 4.06$. The peak in the response spectrum at 60 Hz is due to 60 cycle line noise and the peaks at approximately 165 Hz and 820 Hz correspond to the first and second resonant frequencies, respectively. Figures 4 and 5 show the cross-spectral density function in the form defined by Eq. (4). Figure 6 gives the magnitude of the system frequency response function 1) as measured by the sine dwells and 2) as computed according to Eq. (7). Figure 7 depicts the ordinary coherence function. Note that the db scale of Fig. 6 is

$$\text{db} = 20 \log_{10}(\text{PSD of output/PSD of input})^{1/2}$$

while the scales of Figs. 2 and 3 are

$$\text{db} = 10 \log_{10}(\text{PSD of input}/1\ \text{g}^2/\text{Hz})$$

$$\text{db} = 10 \log_{10}(\text{PSD of output}/1\ \text{g}^2/\text{Hz})$$

These results substantiate the use of spectral relationships and ordinary coherency in computing and in assessing the validity of frequency response functions of single input—single output structural configurations. Note that the magnification factors of Fig. 7 are nearly identical; they deviate somewhat from one another near the fundamental

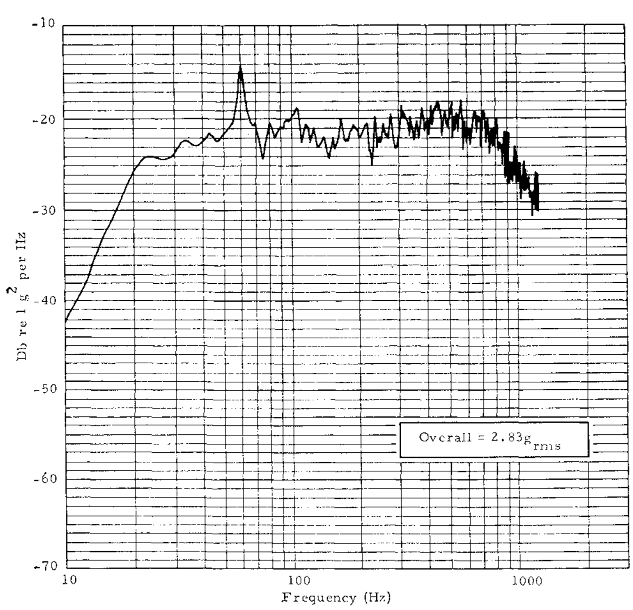

Fig. 2 Acceleration spectral density, $G_x(f)$.

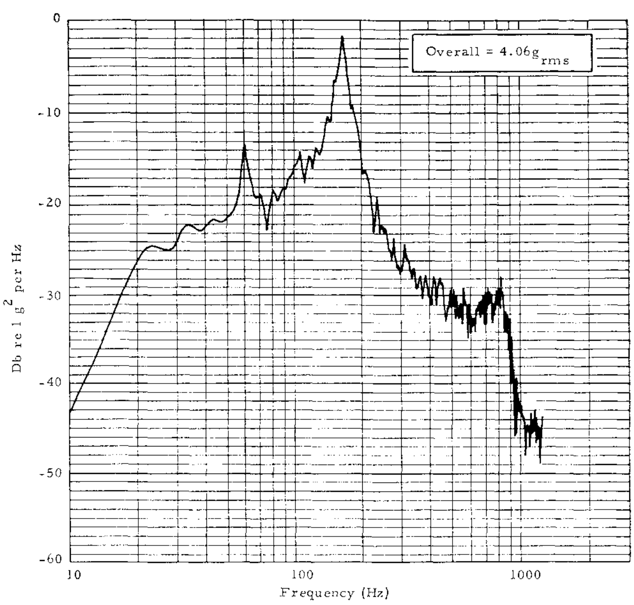

Fig. 3 Acceleration spectral density, $G_y(f)$.

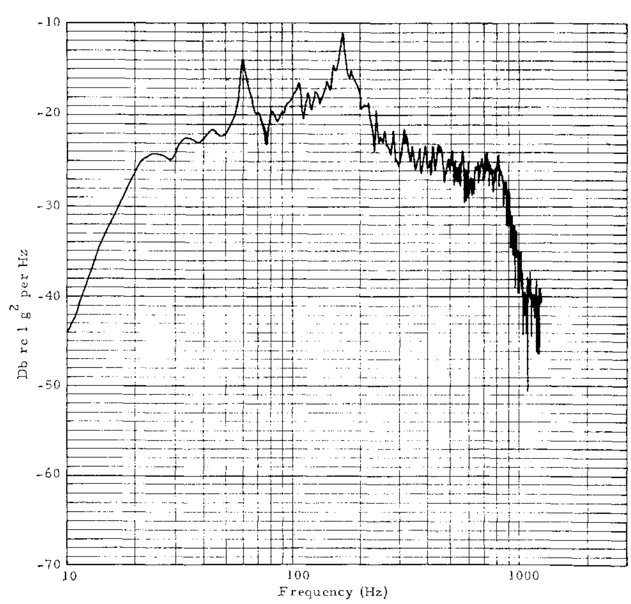

Fig. 4 Cross-power spectral density magnitude, $|G_{xy}(f)|$.

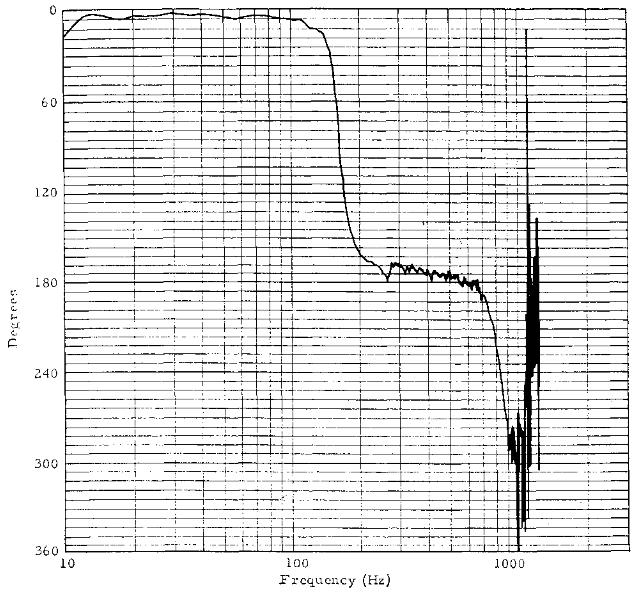

Fig. 5 Phase angle, $\theta_{xy}(f)$.

resonant frequency and beyond 350 Hz. The coherence function, by deviating from unity near 165 Hz and beyond 350 Hz, points out that this behavior should be expected.

To single out a predominant reason for such deviation would require much computational and experimental sleuthing. Such could prove rather expensive and time consuming, and was not attempted here. Possible reasons would include extraneous noise in the input and output signals and/or extraneous noise introduced by computations in the digital processing. Other reasons would be errors induced by signal conditioning equipment, discriminator filters, and possibly tape skew.

Concluding Remarks

Much computational and experimental work is required before coherency becomes an effective analysis tool for multi-input structural configurations. Methods and techniques for detecting and distinguishing between nonlinearities and noise in random signals need to be developed. Although related work[11,12] has been done for structural systems, virtually no use has been made of coherence functions. On the basis of the results shown here, such functions may hold promise in future structural data analysis of random tests and flight programs.

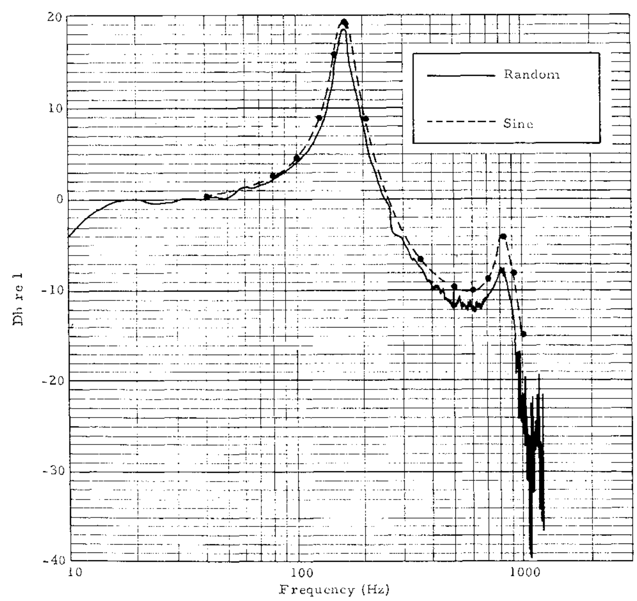

Fig. 6 Magnitude of frequency response function [magnification factor, $|H(f)|$].

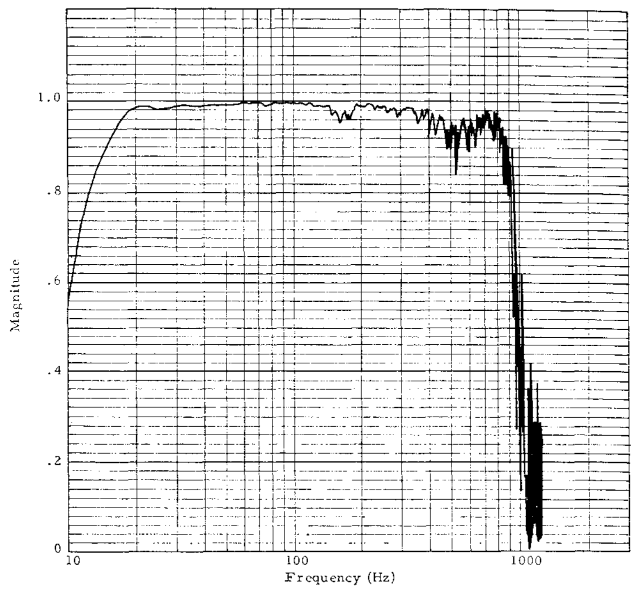

Fig. 7 Ordinary coherence function, $\gamma_{xy}^2(f)$.

References

[1] Bendat, J. S. and Piersol, A. G., *Measurement and Analysis of Random Data,* Wiley, New York, 1966, pp. 105–122.

[2] Goodman, N. R., "Measurement of Matrix Frequency Response Functions and Multiple Coherence Functions," AFFDL-TR-65-66, Wright-Patterson Air Force Base, Ohio, June 1965.

[3] Tick, L. J., "Conditional Spectra, Linear Systems, and Coherency," *Proceedings of the Symposium on Time Series Analysis,* edited by M. Rosenblatt, pp. 197–203.

[4] Akaike, H., "Statistical Measurement of Frequency Response Functions," *Annals of the Institute of Statistical Mathematics,* Supplement III, 1964, pp. 5–17.

[5] Jenkins, G. M., "Cross-Spectral Analysis and the Estimation of Linear Open-Loop Transfer Functions," *Proceedings of the Symposium on Time Series Analysis,* edited by M. Rosenblatt, SIAM Series in Applied Mathematics, Wiley, New York, 1963, pp. 267–278.

[6] Dean, W. C., Enochson, L. D., and Shumway, R. H., "The Coherency Analysis of Seismic Noise," SDLR 155, Air Force Technical Applications Center, Washington, D.C., July 1966.

[7] Fehr, U., Tolman, W. F., and Cralene, R., "Systems and Methods in Statistical Analysis and Reduction of Geoacoustical and Geomagnetic Data," *The Journal of the Acoustical Society of America,* Vol. 42, No. 5, Nov. 1967, pp. 1008–1016.

[8] Sato, H., "The Measurement of Transfer Characteristics of Ground-Structure Systems Using Micro Tremor," *Annals of the Institute of Statistical Mathematics,* Supplement III, 1964, pp. 71–78.

[9] Kaneshige, I., "Frequency Response of an Automobile Engine Mounting," *Annals of the Institute of Statistical Mathematics,* Supplement III, 1964, pp. 49–57.

[10] Enochson, L. D. and Otnes, R. K., *Programming and Analysis for Digital Time Series Data,* Shock and Vibration Monograph No. 3, Defense Documentation Center, Cameron Station, Alexandria, Va.

[11] Ailman, C. M. and Hopkins, A. S., "Narrow Band Cross-Correlation Analysis of Fluctuating Pressures Beneath a Turbulent Boundary Layer," CR-1066, May 1968, NASA.

[12] Trubert, M. R. P., "Response of Elastic Structures to Statistically Correlated Multiple Random Excitations," *The Journal of the Acoustical Society of America,* Vol. 35, No. 7, July 1963, pp. 1009–1022.